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A. NARASINGA RAO, M.A., L.T.

With the co-operation of
V. RAMASWAMI AIYAR, M.A.; S. SIVASANKARANARAYANA PILLAI, M.Sc.;
G. A. SRINIVASAN, M.A., L.T.; B. RAMAMURTI, M.A.;
N. SUNDARAM IYER, M.A.; A. A. KRISHNASWAMI AYYANGAR, M.A.;
C. N. SRINIVASIENGAR, D.Sc.; S. PURUSHOTHAM, M.A.
and others

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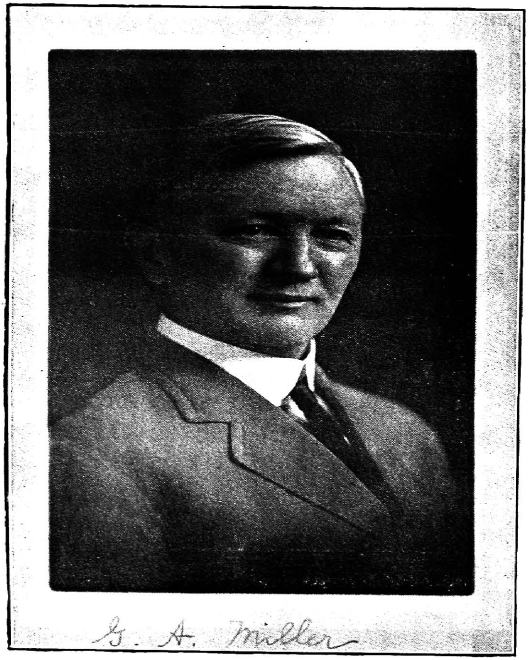

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THE MATHEMATICS STUDENT

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NOTE ON A THEOREM ON HOMOLOGOUS STARS.

By B. S. Madhava Rao,

College of Engineering, Bangalore.

1. This note is concerned with the proof of the well-known theorem that

> *the solutions of Emden's polytropic equation obtained by assuming different central conditions fall into a homologous series.**

The usual proofs assume the series of configurations to be of the same mass, and for such polytropic stars, it is shown that the ratio $\bar{\rho}/\rho_c$ is independent of the central conditions whence it follows that all these stars have the same law of density-distribution.† It has, however, been pointed out by Milne‡ that the concept of "***homologous stars series***" should be generalised so as to include stars of all masses and all radii having the same density-distribution thus leading to a doubly infinite system which might be called a "***homologous family***"

---

* Jeans, *Astronomy and Cosmogony* (1928), p. 71.

† Emden Thermo-dynamik der Himmelskorper (*Ency. Math. Wiss.* (Bd. VI, 2B), p. 408, § 19.

‡ M. N. R. A. S., Nov. 1929, p. 27, § 14.

According to this definition, it is quite easy to see that the solutions of Emden's equation (even when we restrict ourselves to the particular Emden-Eddington solutions without taking into account the generalised Milne-Fowler solutions) with different central conditions form such a family and it is the object of this note to establish that this family is homologous.

2. A homologous series is defined usually to be a series of configurations of constant mass which satisfy Lane's laws, *viz.*, that if a mass of gas be in equilibrium with the values r, ρ, P, T, these values can be changed to r', ρ', P', T' given by

$$r' = \alpha r ; \quad \rho' = \alpha^{-3}\rho ; \quad P' = \alpha^{-4} P ; \quad T' = \alpha^{-1} T$$

and the mass will still be in equilibrium.* In order to prove the theorem in question for the generalised family it is therefore only necessary to use Lane's laws in a form suitable for application to the case of variable mass. Such a form is well-known † and is to the effect that at corresponding points within any number of stars of a homologous family (*corresponding points* being those points whose distances from the centres are the same fractions of the radii) the densities vary as M/r^3, the pressures as M^2/r^4 and the temperatures as $\mu M/r$, where μ is the molecular weight at the point considered.

3. We now proceed with the proof adopting throughout the notation employed by Eddington.‡

The fundamental equation of polytropic gas spheres are

$$dP = -g\rho\, dr \qquad \dots \quad (1)$$

and

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = -4\pi G\rho \qquad \dots \quad (2)$$

---

* Jeans. *Astronomy and Cosmogony*, p. 65.

† Russell-Dugan-Stewart. *Astronomy*, Vol. 2. p. 884.

‡ *Internal Constitution of Stars*, Chap. IV.

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where ϕ is the gravitational potential, and the polytropic equation,

$$P = k\rho^{\gamma} \qquad \ldots \quad (3)$$

where $\gamma = 1 + 1/n$, n being the polytropic index. From equations (2) and (3) we easily deduce the equation

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} + \alpha^2\phi^n = 0 \qquad \ldots \quad (4)$$

where $$\alpha = 4\pi G / \{ (n+1)^k \}^n$$

with the further relations,

$$\rho = \left\{ \frac{\phi}{(n+1)k} \right\}^n \qquad \ldots \quad (5)$$

and $$P = \frac{\rho\phi}{n+1} \qquad \ldots \quad (6)$$

If ϕ_0 be the value of ϕ at the centre, we then substitute

$$\phi = \phi_o u; \quad r = z / \alpha\phi_o^{\frac{1}{2}(n-1)} \qquad \ldots \quad (7)$$

in equation (4) and deduce Emden's equation in the standard form

$$\frac{d^2u}{dz^2} + \frac{2}{z}\frac{du}{dz} + u^n = 0 \qquad \ldots \quad (8)$$

with the central conditions, $u = 1$ and $du / dz = 0$ when $z = 0$.

If we have to consider, now, a solution of Emden's equation (8) with a different central condition, $u = u_o$ and $du / dz = 0$, when $z = 0$ we need only alter the substitutions (7) replacing them by

$$\phi / \phi_o = u / u_o \text{ and } r = z / \alpha(\phi_o / u_o)^{\frac{1}{2}(n-1)} \qquad \ldots \quad (9)$$

which would again transform (4) to the same equation (8), but with different central conditions. Let ρ', P', T', r', M' refer to the polytropic sphere with the central condition $u = u_o$ and the same quantities

without accents for $u = 1$. We then have

$$\rho = \left\{ \frac{\phi_o u}{(n+1)^k} \right\}^n ; \quad \rho' = \left\{ \frac{(\phi_o / u_o) u}{(n+1)k} \right\}^n \qquad \text{from (5)}$$

$$P = \frac{\rho \phi}{n+1} \quad ; \quad P' = \frac{\rho' \phi'}{n+1} \qquad \text{from (6)}$$

where ϕ' stands for $(\phi_o / u_o) u$.

As regards temperature the assumption of the perfect gas equation, $p_G = \frac{\mathscr{R}}{\mu} \rho T$ leads to the relation $T = \frac{\beta \mu \phi}{(n+1)\mathscr{R}}$, where β is the ratio of gas pressure to the whole pressure and $\mathscr{R}$ the universal gas constant. We thus have

$$T = \frac{\beta \mu \phi}{(n+1)\mathscr{R}} ; \quad T' = \frac{\beta \mu \phi'}{(n+1)\mathscr{R}}$$

(assuming β and μ to be the same in both the cases).

Hence $\rho/\rho' = u_o^n$; $P/P' = u_o^{n+1}$; $T/T' = u_o$.

$$\text{Again,} \quad GM = \left(- r^2 \frac{d\phi}{dr} \right)_{u=o} = \frac{1}{a \phi_o^{\frac{1}{2}(n-3)}} \left(- z^2 \frac{du}{dz} \right)_{u=o}$$

$$\text{and,} \quad GM' = \frac{1}{a (\phi_o / u_o)^{\frac{1}{2}(n-3)}} \left(- z^2 \frac{du}{dz} \right)_{u=o}$$

$$\therefore \quad M/M' = u_o^{\frac{1}{2}(n-3)}$$

$$\text{and finally} \quad r'/r = u_o^{\frac{1}{2}(n-1)}.$$

$$\text{Hence,} \left(\frac{M}{r^3} \right) \Big/ \left(\frac{M'}{r'^3} \right) = \left(\frac{M}{M'} \right) \left(\frac{r'}{r} \right)^3 = u_o^{-\frac{1}{2}(n-3)} \cdot u_o^{\frac{3}{2}(n-1)}$$

$$= u_o^n = \left(\frac{\rho}{\rho'} \right).$$

Again $\left(\frac{M^2}{\gamma^4}\right) \Big/ \left(\frac{M'^2}{\gamma'^4}\right) = \left(\frac{M}{M'}\right)^2 \left(\frac{r'}{r}\right)^4 = u_o^{-(n-3)} \cdot u_o^{2(n-1)}$

$$= u_o^{n+1} = \left(\frac{P}{P'}\right)$$

and finally, $\left(\frac{M}{r}\right) \Big/ \left(\frac{M'}{r'}\right) = u_o^{-\frac{1}{2}(n-3)} \cdot u_o^{\frac{1}{2}(n-1)} = u_o = \left(\frac{T}{T'}\right)$

Thus from the generalised form of Lane's laws, it follows immediately that the solutions form a homologous family. We can also easily verify that the ratio $\bar{\rho}/\rho_c$ is the same for all members of the family, for,

$$\bar{\rho} = \quad / \tfrac{4}{3}\pi R^3 = \frac{1}{G}\left(-\frac{1}{4/3\,\pi r^3} \cdot r^2 \frac{d\phi}{dr}\right)_{u=o}$$

$$= \frac{\alpha^2 \phi_o^n}{4\pi G}\left(-\frac{3}{z} \cdot \frac{du}{dz}\right)_{u-o}$$

and, $\qquad \alpha^2 \phi_o^n = 4\pi G \rho_c.$

Thus $\qquad (\bar{\rho}/\bar{\rho}') = u_o^n$ and $(\rho_c/\rho_c') = u_o^n$

which shows $\qquad (\bar{\rho}/\rho_c) = (\bar{\rho}'/\rho_o').$

ON THE GREATEST PRIME FACTOR OF A CERTAIN PRODUCT.

By S. M. Shah,
Muslim University, Aligarh.

1. Let P_x denote the greatest prime factor of the product

$$f(x) = (1 + 1^2)(1 + 2^2) \ldots (1 + x^2).$$

It is known that*
$$\lim_{x \to \infty} P_x / x = \infty.$$

Dr. S. D. Chowla has proved a better result,† namely :

If $\varepsilon > 0$, *then we can find a number* ξ *such that*

$$P_x > (2 - \varepsilon)\, x \log x$$

for all integers $x > \xi = \xi(\varepsilon)$.

In this note, I prove that

$$P_x > 2x \log x - 2Bx$$

for all integers $x > x_0$ *where* $B \geqslant \frac{3 \log 2}{2} + 11 + 2C$

and
$$C = \lim_{x \to \infty} \left\{ \sum_{\substack{p \equiv 1 \,(mod\, 4) \\ p \leqslant x}} \frac{\log p}{p} - \tfrac{1}{2} \log x \right\}$$

and x_0 *is a constant depending on* B.

---

* Landau *Handbuch*, 559—561.

† *Journal of the Ind. Math. Soc.*, Vol. XVIII (1929).

2. If $f(x)$ has no prime factor exceeding $2x \log x - 2Bx$, we get

$$\log f(x) = \sum_{p} \sum_{m=1}^{\infty} \{ \theta(p^m) \log p \}$$

where p runs through all primes not exceeding $2x \log x - 2Bx$ and $\theta(n)$ is the number of numbers for which

$$y^2 + 1 \equiv 0 \pmod{n} \qquad (y = 1, 2 \ldots x)$$

Now

$$\sum_{p} \sum_{m=1}^{\infty} \{ \theta(p^m) \log p \}$$

$$= \sum_{\substack{p \\ 2 \leqslant p < 2x}} \sum_{m=1}^{\infty} \{ \theta(p^m) \log p \} + \sum_{\substack{p \\ 2x < p \leqslant 2x \log x - 2Bx}} \sum_{m=1}^{\infty} \{ \theta(p^m) \log p \} = \Sigma_1 + \Sigma_2$$

Let $x > x_0$ then*

$$\Sigma_1 \leqslant \log 2 \left(\frac{x+1}{2}\right) + \sum_{\substack{p \equiv 1 \pmod 4 \\ p < 2x}} \log p \left\{ \left(\frac{2x}{p} + 2\right) + \left(\frac{2x}{p^2} + 2\right) + \ldots \left(\frac{2x}{p^m} + 2\right) \right\}$$

where m is the greatest integer for which $p^m \leqslant 1 + x^2$ and hence $m \leqslant \frac{\log(1 + x^2)}{\log p}$. It follows that

$$\Sigma_1 \leqslant \frac{\log 2}{2} x + O(1) + 2x \sum_{\substack{p \equiv 1 \pmod 4 \\ p < 2x}} \frac{\log p}{p-1} + 2 \sum_{\substack{p \equiv 1 \pmod 4 \\ p < 2x}} m \log p$$

Now

$$\sum_{\substack{p \equiv 1 \pmod 4 \\ p \leqslant x}} \frac{\log p}{p} = \tfrac{1}{2} \log x + C + o(1)$$

---

* See Chowla. *loc. cit*

$$\therefore \sum_{\substack{p\equiv 1 \pmod 4 \\ p<2x}} \frac{\log p}{p-1} = \sum_{\substack{p\equiv 1 \pmod 4 \\ p<2x}} \frac{\log p}{p(p-1)} + \sum_{\substack{p\equiv 1 \pmod 4 \\ p<2x}} \frac{\log p}{p}$$

$$< 2 + (\tfrac{1}{2}\log(2x) + C + o(1))$$

$$\therefore \quad \Sigma_1 \leqslant \frac{\log 2}{2} x + 2x \{ \tfrac{1}{2}\log x + 2 + \tfrac{1}{2}\log 2 + C \}$$

$$+ 2\log(1+x^2)\left\{ \frac{1}{2}\frac{2x}{\log(2x)} + o\left(\frac{x}{\log x}\right) \right\} + o(x)$$

$$= x\log x + \left\{ \frac{3\log 2}{2} + 8 + 2C + o(1) \right\} x$$

Also*

$$\Sigma_2 \leqslant \quad \Sigma \quad (\log p) \quad \left(\begin{array}{c} p\equiv 1 \pmod 4 \\ 2x<p\leqslant 2x\log x - 2Bx \end{array} \right)$$

$$< \quad \Sigma \quad (\log p) \quad \left(\begin{array}{c} p\equiv 1 \pmod 4 \\ p\leqslant 2x\log x - 2Bx \end{array} \right)$$

$$= x\log x - Bx + O(x\log x \; e^{-a_2\sqrt{\log(x\log x)}})$$

$$= x\log x - Bx + o(x)$$

$$\log f(x) \leqslant 2x\log x + x\left\{ \frac{3\log 2}{2} + 8 + 2C - B \right\} + o(x)$$

$$< 2x\log x - 3x + o(x).$$

But $\qquad \log f(x) = 2x\log x - 2x + o(x).$

Hence we have a contradiction, and therefore our result is proved.

---

* Chowla, *loc. cit.*

A GRAPHICAL TREATMENT OF CERTAIN INEQUALITIES.

By M. N. Narasimha Iyengar,
Bangalore.

In this paper I propose to discuss certain inequalities which are suggested by the convexity or concavity of certain well-known curves. While the analytical proof is rightly considered to be far more satisfactory than an appeal to the figure, the latter has a higher intuitional value both for indicating the existence of certain inequalities, and for fixing them in the memory. Besides, the figure often suggests an analytical proof as well.

2. Let $f(x)$ be a real function of a real variable which possesses derivatives of the first two orders in an interval (a, b). If for all x in (a, b), we have $\frac{d^2 f(x)}{dx^2} > 0$, *the function is said to be convex in the interval.* On the other hand if $\frac{d^2 f(x)}{dx^2} < 0$, in (a, b), then it is *concave.*

The following simple properties of convex* (and concave) functions which are all that will be required for this discussion, will be obvious from the graph :

* [The study of *convex functions* was initiated by J. L. W. V. Jensen in an important paper published in the *Acta Mathematica* (Vol. 30). A real, one valued function $f(x)$ is, according to Jensen, *convex* in an interval (a, b) if for every x and y in the interval $f(x) + f(y) \geqslant 2f\left(\frac{x+y}{2}\right)$. It is shown that a convex function which is bounded above in an interval is continuous in the interval, and has both a right-hand and a left-hand derivate. When the function possesses derivates of the first two orders, (which implies the continuity of the function and its first derivate) the graph has a tangent continuously turning in the anti-clockwise direction as x increases, and is convex in the sense of this paper. The graph will be convex as seen from below. Readers interested in inequalities (and in analysis generally) will find much that is valuable and interesting in Polya and Szego : "*Aufgaben und Lehrsatze aus der Analysis,*" published by Julius Springer, Berlin.—Ed.]

If $f(x)$ is convex in the interval (a, b), and $a > b$ then

$$f'(a) > \frac{f(a) - f(b)}{a - b} > f'(b). \qquad \ldots \quad (2.1)$$

Geometrically, this asserts that the slopes of the tangent at A $(a, f(a))$ > the slope of the chord AB > the slope of the angent at B $(b, f(b))$ if $a > b$. This follows from the fact that the tangent has a counter-clockwise rotation as the abscissa of the point of contact increases. Analytically, using the theorem of mean value, (2.1) is equivalent to $f'(a) > f'(\xi) > f'(b)$ where $a > \xi > b$, a result which is a consequence of $f'(x)$ being an increasing function of x, that is of $(f''(x) > 0)$.

Another inequality which follows by a repeated application of (2.1) is :

if $a > b > c$, and $f(x)$ is convex in the interval (a, c) then

$$\frac{f(a) - f(b)}{a - b} > \frac{f(b) - f(c)}{b - c}. \qquad \ldots \quad (2.2)$$

Lastly consider a set of points A_1, A_2, A_3 ... ... A_n whose x-co-ordinates $a_1, a_2, a_3, \ldots a_n$ form an increasing sequence, at which positive masses $p_1, p_2, p_3 \ldots p_n$ are located. The centre of gravi:y of these is given by

$$\bar{x} = \sum_r p_r a_r / \sum_r p_r; \qquad \bar{y} = \sum_r p_r f(a_r) / \sum_r p_r \qquad \ldots \quad (2.3)$$

and lies within the convex region bounded by the curve and the chord $A_1 A_n$, that is ***above*** the curve. Hence*

for a convex curve $\bar{y} > f(\bar{x})$ where $\bar{x}$ and $\bar{y}$ are given by (2.3). ... (2.4)

---

* [For $f(a_r) = f(\bar{x}) + (a_r - \bar{x}) f'(\bar{x}) + \frac{(a_r - \bar{x})^2}{2!} f''(\xi_r)$.

Hence $\Sigma p_r f(a_r) = f(\bar{x}) \Sigma p_r + \frac{1}{2} \Sigma p_r (a_r - \bar{x})^2 f''(\xi_r) > f(\bar{x}) \quad p_r$

which proves (2.4) *vide* Polya and Szego : Vol. I, Solutionsıto Questions 76 and 74 Abschnitt II. The solution to the latter gives a proof which does not assume the existence of $f''(x)$, but only the continuity of $f'(x)$.—ED.]

All the inequalities mentioned above will be reversed if the function is concave instead of convex. ... (2.5)

It should be specially noted that the inequalities do not assume that either $f(x)$ or $f'(x)$ is positive, and hold equally whether the curve is above or below the x-axis and whether the function is increasing or decreasing.

3. Let $f(x) = x^p$ so that $f(x)$ is real, positive, and continuous in $x > \varepsilon > 0$. Since $f''(x) = p(p-1)x^{p-2}$ the function is convex or concave in the interval according as p does not, or does lie in the interval (0, 1). From (2.1) we get the important inequality

$$pa^{p-1}(a-b) > a^p - b^p > pb^{p-1} \qquad \left(\begin{matrix} a > b \\ p(p-1) > 0 \end{matrix}\right) \quad \ldots \quad (3.1)$$

Again (2.2) gives

$$\frac{a^p - b^p}{a-b} > \frac{b^p - c^p}{b-c} \text{ if } a > b > c; \text{ and } p(p-1) > 0. \ \ldots \quad (3.2)$$

Here put $p = m/n$ and replace a, b, c by a^n, b^n, c^n where m and n are positive and $m > n$. With these values (3.2) is valid and we have

$$\frac{a^m - b^m}{a^n - b^n} > \frac{b^m - c^m}{b^n - c^n}$$

which may be rewritten

$$\Sigma\,(a^m b^n - a^n b^m) > 0^* \qquad (a > b > c;\ m > n > 0). \ \ldots \quad (3.3)$$

The first member of (3.3) is unaltered by a cyclic interchange of a, b, c, and hence *the result is valid also when* $b > c > a$ or $c > a > b$. That such cyclical changes are permissible we shall indicate by writing

$$a > b > c \text{ cycl.}$$

On the other hand the inequality will be reversed when $c > b > a$ or $b > a > c$ or $a > c > b$, since the first member of (3.3) alters its sign for an interchange of any two of the letters a, b, c.

---

* This inequality is given by Segar in the *Messenger of Mathematics*, Vol. 20. In Vol. 22 it is asserted that (3.3) also holds good even if m is negative *provided it is numerically greater than* $n > 0$. It will be seen from the above that if $n > 0$ and m is negative (so that $p < 0$) the argument is still valid and that the italicised proviso is unnecessary.

4. Next let us take $f(x) = \log x$ so that $f''(x) < 0$, and hence the curve *concave*.

From the formulae (2.1) and (2.5) we have

$$\frac{1}{a} < \frac{\log a - \log b}{a - b} < \frac{1}{b} \qquad (a > b)$$

which, for $a = n + 1$, $b = n > 0$ leads to the important inequality

$$\frac{1}{n+1} < \log\left(1 + \frac{1}{n}\right) < \frac{1}{n}. \qquad \ldots \quad (4.1)$$

Again (2.2) and (2.5) give

$$\frac{\log a - \log b}{a - b} < \frac{\log b - \log c}{b - c} \qquad (a > b > c \text{ cycl.} > 0)$$

which may be re-written

$$(a/b)^c\,(b/c)^a\,(c/a)^b > 1 \qquad (a > b > c \text{ cycl.} > 0) \ldots \quad (4.2)$$

an inequality also due to Segar. This inequality is reversed if $b > a > c$ cycl.

Lastly, using (2.4) and (2.5) we have on simplification

$$\left(a_1^{p_1}\, a_2^{p_2} \ldots \ldots a_n^{p_n}\right)^{1/(p_1 + \cdots p_n)} < \frac{p_1 a_1 + \ldots p_n a_n}{p_1 + \ldots p_n}. \quad \ldots \quad (4.3)$$

If we put $p_1 = p_2 \ldots = p_n = p$, this gives the familiar result

$$\sqrt[n]{a_1 a_2 \ldots \ldots a_n} < (a_1 + \ldots a_n)/n. \qquad \ldots \quad (4.4)$$

More generally taking $f(x) = x^p \log x$, which is convex for $p > 1$, $x > 1$ and using (2.2) we have on putting $p = m/n$, $(m > n > 0)$

$$a^{a^m(b^n - c^n)} \, . \, b^{b^m(c - a)^n} \, . \, c^{c^m(a^n - b^n)} < 1 \qquad (a > b > c \text{ cycl.} > 1). \quad (4.5)$$

5. Next, let $f(x) = a^x$ where $x > 0$, $a \neq 1$, so that the curve is *convex*. Here we have the slope of a chord LQ less than that of LP

where L, Q, P are points on the curve corresponding to $x = 0, q, p$, and $0 < q < p$. Hence we have the well-known inequality

$$\frac{a^p - 1}{p} > \frac{a^q - 1}{q} \qquad (p > q > 0,\ a > 0) \quad \ldots \quad (5.1)$$

which reduces to an equality when $a = 1$.

Another result deduced by using (2.1) with $a = n,\ b = n - 1$, is

$$a^a > e^{a-1} > a,$$

while the application of (2.2) gives

$$a^p q + a^q r + a^r p > a^p r + a^q p + a^r q \qquad (p > q > r > \text{cycl.} > 0).$$

Lastly, taking $f(x) = x^x$, the function is continuous and convex in $x > \varepsilon > 0$ since $f''(x) = x^{x-1}\{1 + x(1 + \log x)^2\}$. By using the results (2.1), (2.2) and (2.3) we obtain the following results as may be easily verified:

$$a^a (a - b)(1 + \log a) > a^a - b^b > b^b (a - b)(1 + \log b) \qquad (a > b) \quad \ldots \quad (5.3)$$

$$a^a (b - c) + b^b (c - a) + c^c (a - b) > 0 \quad (a > b > c > 0 \text{ cycl.}) \ldots \quad (5.4)$$

$$\left(\frac{pa^a + qb^b + \ldots\ldots}{p + q + \ldots\ldots}\right)^{p+q+\ldots} > \left(\frac{pa + qb + \ldots\ldots}{p + q + \ldots\ldots}\right)^{pa+qb+\ldots} \quad \ldots \quad (5.5)$$

$p, q, r, \ldots\ldots$ being positive.

As a special case of (5.5) we have

$$\left(\frac{a^a + b^b + \ldots\ldots}{n}\right)^n > \left(\frac{a + b + \ldots\ldots}{n}\right)^{a+b+\ldots}. \quad \ldots \quad (5.6)$$

GLEANINGS.

9. Even as the finite encloses an infinite series
   And in the unlimited, limits appear,
So the soul of immensity dwells in minutia
   And in narrowest limits no limits inhere.
What joy to discern the minute in infinity!
   The vast to perceive in the small, what divinity.

JACQUES BERNOULLI: *On Infinite Series* (Trans. by Helen M. Walker, From D. E. Smith's *Source Book of Mathematics*)

A SEQUEL TO HAMILTON'S THEOREM.

By A. A. Krishnaswami Ayyangar, M.A., L.T.,
Maharaja's College, Mysore.

1. It is the object of this note to prove a number of useful properties connected with conjugate diameters of a conic by simple geometrical methods and to show how they are all related in a natural manner to a theorem due to Hamilton.

2. CB is a fixed diameter of a given circle, A a fixed point in CB produced. Through A draw any line meeting the circle in D and E. Join CD and produce it either way to F making CF = AE; the locus of F is the ellipse

$$\frac{x^2}{AC^2} + \frac{y^2}{AB^2} = 1.$$

This is the theorem attributed to Hamilton by Casey in his ***Treatise on Analytical Geometry***.* The theorem is true even when A is between C and B. Strangely, however, on p. 253 of his ***Treatise***, Casey gives the same theorem, erroneously stating that the locus is a hyperbola.†

The proof of the theorem is easy and need not detain us. We shall proceed directly to a modification of the theorem and to its applications.

Theorem I. *P is a fixed point on the circle with CB as diameter and A is a fixed point on CB or CB produced. DE is a chord of the circle through A. PD is joined and on it produced either way a length PQ is taken equal to AE. The locus of Q is the ellipse whose semi-axes are equal to AC, AB.*

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* *Vide* p. 205 Casey's *Analytical Geometry*, 2nd Edition, 1893.

† Mr. V. Ramaswami Iyer draws attention to this error in his question 1595 (*J. I. M. S.*, Vol. XIX, Notes and Questions, p. *80*).

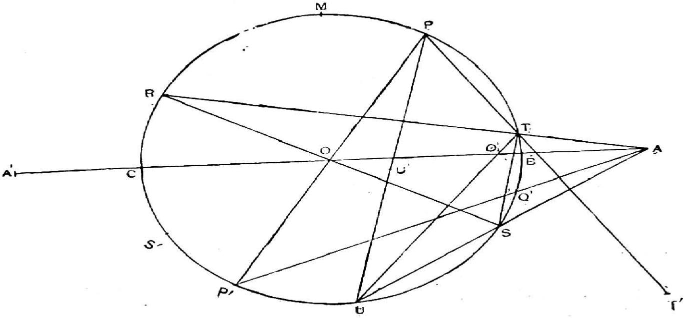
For, the angle between PQ and CF is constant, and the locus of Q is the locus of F in Hamilton's theorem turned through a constant angle (equal to $P\hat{D}C$) about C and given a translation represented by the vector CP, so that C, P are the corresponding points associated with the two loci.

This theorem enables us to find the length of a central radius-vector of an ellipse in any given direction, and is useful in proving results relating to the lengths of diameters of a conic. For instance, we may deduce that the sum of the squares of the reciprocals of perpendicular diameters is constant.

It should be noted that the ellipse passes through A since the length of the radius-vector in the direction PA is equal to PA. The lengths of the radii-vectors in the directions PB, PC are equal to AC, AB respectively.

3. We now study the directions and magnitudes of a pair of conjugate diameters of the locus.

THEOREM II. *CB is a fixed diameter of a circle centre O, and A a fixed point in CB or CB produced. RS is any variable diameter; AR, AS meet the circle again in T, U. On PT, PU are taken respectively T′, U′ such that PT′ = AR and PU′ = AS. Then PT′, PU′ are a pair of conjugate semi-diameters* in position and magnitude.



For,* (i) T′, U′ are points on the ellipse of Theorem I.

and (ii) PT′ . PU′ sin ΓPU = AR . AS . sin ASΓ

= AR . AT

= AC . AB.

Conversely, we can show that if PT′, PU′ be a pair of conjugate diameters which cut the circle again in T, U and AT, AU cut the circle again in R, S, then RS is a diameter of the circle.

Cor. 1. If P′ be diametrically opposite to P on the circle, and AP′ cut the circle again at Q′, then PQ′ is in the direction conjugate to PA, and the length of the conjugate diameter in this direction is 2 AP′.

This property is used by Mannheim† to find the magnitude and direction of the axes of a conic, given two conjugate diameters in position and magnitude. Incidentally, it suggests how we can get the circle in Hamilton's theorem, starting from the ellipse.

Cor. 2. The directions PC, PB are those of a pair of conjugate diameters. Since they include a right angle, they are the directions of the semi-axes, the corresponding lengths being AB, AC. From this property again, we can derive as follows a construction for the circle given the ellipse :

> Let P be the centre and A any point of the ellipse. With centre A and radius equal to the semi-minor axis draw an arc to cut the major-axis in B. Join AB and let it cut the other axis in C. The circle PBC is the required circle. It can be easily seen that AC is the semi-major axis, and the ellipse of theorem I is easily identified with the given one.

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* If PU′ be not conjugate to PT′, let PU″ be the semi-diameter conjugate to PT′; then, since △ PU′T′ = △ PU″T″, we have U′U″ is parallel to PT′ and is therefore a tangent at U″, which is impossible.

† *Vide* Casey *l. c.*, pp. 210, 211 where an incomplete proof is given of Mannheim's construction, incomplete in the sense that it is not proved there that OA, OB are the directions of the axes. *Vide* Question 1594 in *Jour. Ind. Math. Soc.*, Vol. XIX, pp. 79, 80.

***Cor.* 3.** If RS be a diameter perpendicular to CB, then AR = AS and PT′, PU′ become equi-conjugate semi-diameters.

Cor. 4. As the diameter RS varies, TU meets CB in a fixed point O′; for applying Desargues' theorem to the circle and the line pairs which pass through the four points R, S, T, U, we find that A is a double point of the involution to which (C, B), (O, O′) belong. Since A is real, and O lies between C and B, O′ must also lie between C and B, *i.e.*, within the circle.

4. Conjugate Diameters containing a given acute angle α.

Following the notation of Theorem II, if PT′, PU′ be a pair of conjugate diameters, we have if α is the acute angle between the conjugate diameters

$$\alpha = T\hat{S}A = A\hat{S}A' \sim A'\hat{S}T = A\hat{S}A' \sim \pi/2$$

(A′ being the image of A in O); for $A'\hat{S}T = A\hat{T}S = \pi/2$, A′S being parallel to AR.

Thus $A\hat{S}A'$ is known and hence S can be fixed on the circle and the construction easily completed. There are evidently two pairs of conjugate diameters containing the given angles, and these correspond to the two points S, S′ in which the segment on AA′ containing $\pi/2 \pm \alpha$ (the upper or lower sign to be taken according as A lies in CB produced or CB itself) cuts the circle on CB as diameter.*

When these two points coincide, the segment touches the circle and the point of contact, by symmetry, is an extremity, say M of the diameter perpendicular to CB, and we get the case of equi-conjugate diameters, (*vide* Th II, *Cor.* 3).

By elementary geometry, M is the point on the circle at which AA′ subtends the maximum or minimum value of $(\pi/2 \mp \alpha)$, according as A lies on CB or CB produced; consequently the acute angle 'α'

* An exceptional case occurs when $\alpha = \pi/2$, when the segment reduces to the straight line AA′. This leads to the construction of major and minor axes,

between the conjugate diameters is always minimum when they are equal.

Hence, we have the well-known theorem :

The acute angle between a pair of equi-conjugate diameters is less than the angle between any other pair of conjugate diameters.

It is easily verified that $\tan \frac{1}{2}(AMA' \sim \pi/2) = \sqrt{1-e^2}$ where e is the eccentricity and the acute angle between the equi-conjugate diameters is $AMA' \sim \pi/2$. This angle increases as e diminishes and *vice versa.*

Hence, in two conics, if the acute angle between a pair of conjugate diameters in the one be equal to that between the pair of equi-conjugate diameters in the other, the eccentricity of the conic is greater than that of the other, and conversely.

5. A System of conics having two pairs of conjugate diameters given in position.

Let (PT, PU), (PT_1, PU_1) be the directions of two pairs of conjugate diameters. Draw any circle with centre O and radius OP to cut them in (T, U), (T_1, U_1) and let TU, T_1U_1 meet in O′. Join OO′ to cut the circle in C, B. On CB, or CB produced find A* the double point of the involution determined by (O, O′), (B, C). If A is real we obtain, by the construction of Theorem I, the ellipse which has the given pairs of lines through P as conjugate diameters.

For, as in Theorem II, if AT, AU cut the circle again in R, S, it is easy to see, by Desargues' theorem, that RS passes through O and therefore PT, PU are the directions of conjugate diameters ; similarly PT_1, PU_1.

It is not difficult to verify that the configuration (excluding P) of points and straight lines in the above construction, (*viz.*, the points

* A is real or imaginary according as O′ is within or outside the circle. When O′ is within the circle the tangents from O′ to the circle are imaginary and likewise the double lines of the involutions to which (PT, PU), (PT_1, PU_1) belong.

O, O′, T, U, T_1, U , A and the lines connecting them) is invariable in species. So the ***ellipses*** obtained by varying the distance OP ***are all similar and similarly situated***, while the same ellipse results by varying only the position of O, keeping OP constant. This is obvious otherwise from the fact that the imaginary asymptotes are fixed in direction, being the double lines of the involution to which the given pairs of lines belong.

We may therefore enunciate the following theorem :

THEOREM III. ***If two pairs of lines through a given point belong to an involution with imaginary double lines, there exists a system of homothetic ellipses having conjugate diameters along the given lines.***

The utility of this theorem lies in projecting orthogonally two given angles into right angles. For we have only to construct an ellipse which has two pairs of conjugate diameters in the directions of the arms of the given angles and project the conic orthogonally into a circle.

The construction of a real ellipse is possible only when the pairs of lines through any point parallel to the arms of the given angles are distinct and belong to an involution pencil with imaginary double lines.

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10. Who made the spider parallels design,
Sure as Demoivre, without rule or sign.

POPE : *Essay on Man.*

11. Cases could be cited where Physicists have been led astray through inattention to mathematical rigour ; but these are rare compared with the Mathematician's misadventures through lack of physical insight.

EDDINGTON : *Internal Constitution of the Stars.*

12. I think the idea that the purpose of study is to arrive at a string of proofs of propositions is a little overdone even in Pure Mathematics...... We might indeed say that whereas for the mathematician insight is one of the tools, and proof the finished product, for the physicist proof is one of the tools and insight the finished product.

EDDINGTON :*Internal Constitution of the Stars.*

NOTES AND DISCUSSIONS.

The Editor welcomes for publication under this heading, brief discussions of interesting problems, critical comments, and suggestions likely to be helpful in the class-room.

Note on* Mr. M. R. Paranjpe's Report on the use of the Mother-tongue as the Medium of Instruction and Examination in certain subjects for the Matriculation Examination of the Bombay University

In this report, Mr. Paranjpe examines whether the standard of answers by students has improved by the scheme of vernacularisation and whether it has lowered the standard of English. The data are taken from the tabulated marks awarded to eleven thousand candidates who appeared for the Matriculation examination of the Bombay University in 1929. The marks obtained by the students in History, in History and Geography together, and in English were considered and the following frequency tables were formed :

(1) the marks in History and Geography of those candidates who answered the History paper in vernacular ;

(2) the marks in History and Geography of those candidates who answered the History paper in English ;

(3) the marks in History of those who answered the paper in English ;

(4) the marks in History of those who answered the paper in the vernacular ;

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* Education, India, *Occasional Reports*, No. 16, 1932.

"A Report on the use of the Mother-tongue as the medium of instruction and examination in certain subjects for the Matriculation examination of the Bombay University, by M. R. Paranjpe, M.A., B.Sc., Poona, Fellow of the University of Bombay."

(5) the marks in English of those who answered the history paper in vernacular ;

(6) the marks in English of those who answered the History paper in English.

eparate tables of distribution for the thirteen centres, as also for all centres together were framed. "Medians" were calculated for each distribution by the use of the interpolation formula. These medians were in each case compared to see whether the "Mother-tongue group" derived any advantage over the "English group" Then the difference of the "English medians" of the "Vernacular group" and of the "English group" in each centre was correlated with the extent of vernacularisation in the centre, to find out whether vernacularisation has caused any deterioration in the standard of English.

From an analysis of the data the following conclusions were drawn :—

(1) The "Mother-tongue group" has scored better than the "English group" in the History paper.

(2) There is urgent need for extending the scheme of vernacularisation to Geography also.

(3) There is no marked correlation between the extent of vernacularisation and the weakness in English of the "Mother-tongue group."

The object of this note is to apply more scientific statistical methods to test the conclusions reached. To find the average of each distribution the "Median" was used instead of the "Arithmetic mean" on the simple ground that the latter involved a great deal of labour in the calculations. The difference between the median values of those who answered in vernacular and those who answered in English was found for each centre and for all the centres together. Except for two or three centres the difference was very high only in the case of the History paper and moderately high in the case of History and Geography together, and this difference was to the advantage of those who answered in

150017

vernacular. Hence it was concluded that vernacularisation had a very good effect in raising the standard of the students. It is, however, sometimes risky to draw conclusions without finding the probability of this difference Hence a more reliable method would be to find the Arithmetic means and use Dr. Fisher's method for finding the significance of the mean. This will give a probability of the advantage of vernacularisation. Moreover, without having recourse to the method of multiplying the medians of History by 2 to make them comparable with the medians of History and Geography together we can find out the probabilities in each case, and find whether the advantage is greater in the case of History alone or History and Geography together.

Similarly, the correlation between the extent of vernacularisation and the weakness in English in each centre and its significance, could be better studied by actually calculating the co-efficient of correlation and its probable error.

The following table gives the means (Arithmetic) in History and in History and Geography for the different centres :

Centre.	History means.			History and Geography means.		
	M. T. Group.	English Group	Difference.	M. T. Group.	English Group.	Difference.
1. Bombay ...	34.3	28.9	5.4	63.7	59.5	4.2
2. Poona ...	35.4	30.3	5.1	61.3	57.8	3.5
3. Ahmedabad...	34.0	27.8	6.2	62.5	55.5	7.0
4. Baroda ...	35.0	30.8	4.2	64.3	60.8	3.5
5 Rajkot ...	33.5	29.7	3.8	60.9	58.4	2.5
6. Bhavanagar ..	32.2	31.2	1.0	57.1	59.2	−2.1
7. Surat ...	34.5	32.9	1.6	64.0	62.2	1.8
8. Sangli ...	32.1	32.7	−0.6	59.5	59.2	0.3
9. Kolhapur ...	32.6	28.9	3.7	63.1	59.1	4.0
10. Dharwar ...	34.1	33.1	1.0	62.0	61.7	0.3
11. Nasik ...	33.5	29.8	3.7	62.3	58.0	4.3
12. Karachi ...	30.3	32.9	−2.6	58.8	61.1	−2.3
13. Hyderabad ...	27.7	33.1	−5.4	57.4	60.6	−3.2
14. All Centres...	34.1	− 29.9	4.2	61.6	59.3	2.3

NOTE.—The History paper has a maximum of 75 marks and History and Geography together a maximum of 150 marks.

Now, using Dr. Fisher's method of finding the significance of the means, we find that the Arithmetic mean of the differences in column four is given by

$$\bar{X} = \frac{31.3}{14} = 2.236.$$

The S. D. of these differences is $S = \sqrt{\frac{143.018}{13}} = \sqrt{11.001}$

$$\frac{S}{\sqrt{N}} = \sqrt{\frac{11.001}{14}} = \sqrt{0.786} = 0.887$$

$$\therefore \quad t = \frac{\bar{X}\ \sqrt{N}}{S} = \frac{2.36}{.887} = 2.525.$$

Referring to the tables, we find P = 0.0275 ; that is, the probability is only about 28 out of 1000 that the difference will exceed 2.525 by chance. Hence the difference between the two means is very significant

Similarly in the case of difference in column 7.

$$\bar{X} = \frac{26.1}{14} = 1.86$$

$$S = \sqrt{\frac{110.618}{13}} = \sqrt{8.509}$$

$$\frac{S}{\sqrt{N}} = \sqrt{\frac{8.509}{14}} = \sqrt{0.608} = 0.78 \text{ nearly.}$$

$$\varepsilon = \frac{1.86}{0.78} = 2.385 \quad \text{and} \quad P = 0.036.$$

Hence the difference between the two means is again very significant.

Both these results show that the students who answered their History papers in the vernacular had decided advantage over those who answered it in English. Incidentally, we find that this advantage appears to be greater in History than in the History and Geography papers together ; and all had necessarily to answer Geography in English. Of course the same conclusions were arrived at by Mr. Paranjpe

In the frequency tables of marks in English the first " class interval " is taken as 0 to 50, whereas the others are only 10 marks. Hence it is not proper to take the frequency of that group in the middle of the class interval in calculating the A. M. This did not however come in the way, because the calculation of the median requires only that group in which it is situated. Hence it is not possible to find the difference of the Arithmetic means of the two groups and find the co-efficient of correlation and its probable error. But here also the conclusions arrived at seem to be sound.

University of Madras, Madras.

N. SUNDARARAMA SASTRI, M.A., *Research Student.*

The Graph of $y = a \cos x + b \sin x$.

1. Let $x'ox$ and $y'oy$ be a pair of rectangular axes in the plane of the paper, and let

$$y = f(x), \qquad (-\pi \leqslant x \leqslant \pi);$$

be the equation of a curve in the plane.

If the paper with the curve be rolled up so as to take the form of a right-circular cylinder of unit radius, the portion of the x-axis between $x = -\pi$ and $x = \pi$ being turned into a circle, then, taking C — the centre of the circle ($x'ox$), as the origin, CO as the axis of X, CA, a line perpendicular to CO in the plane of the circle, as the axis of Y: and a line CZ perpendicular to the plane of the circle as the axis of Z, any point P (x, y) in the xoy plane goes over into a point $(\cos x, \sin x, y)$ in the three dimensional space.

The equations to the rolled up curve $y = f(x)$ are

$$X^2 + Y^2 = 1, \qquad Z = y = f(\cos^{-1} X) = f(\sin^{-1} Y),$$

2. In the case of the curve $y = a \cos x + b \sin x$, we get the equations:

$$X^2 + Y^2 = 1, \qquad Z = aX + bY.$$

This being a plane section of the cylinder represents an ellipse. It is thus seen that the curve $y = a \cos x + b \sin x$ rolls up into a plane ellipse.

Since any plane $Z = aX + bY$ may by a suitable rotation about the Z axis be reduced to the form $Z = cX$, it follows that

$$a \cos x + b \sin x = c \cos (x - \alpha)$$

for suitable values of c and α. It may be noted that this argument does not assume a knowledge of the addition theorem on which the usual proofs are based, but uses only the definitions of $\cos x$ and $\sin x$ and the symmetry of the circle. The semi-major axis of this ellipse is $\sqrt{(1 + a^2 + b^2)}$, and the semi-minor axis is unity. The ends of the axes are the points

$$(\cos \phi,\ \sin \phi,\ \sqrt{a^2 + b^2}), \qquad (-\cos \phi,\ -\sin \phi,\ -\sqrt{a^2 + b^2});$$

$$(-\sin \phi,\ \cos \phi,\ 0), \qquad (\sin \phi,\ -\cos \phi,\ 0)$$

where $\phi = \tan^{-1} \frac{b}{a}$, so that $\sin \phi = \frac{b}{\sqrt{a^2 + b^2}}$, $\cos \phi = \frac{a}{\sqrt{a^2 + b^2}}$.

The eccentricity is $e = \sqrt{a^2 + b^2} / \sqrt{1 + a^2 + b^2}$.

The real foci are

$$\left(\frac{a}{\sqrt{1 + a^2 + b^2}},\ \frac{b}{\sqrt{1 + a^2 + b^2}},\ \frac{a^2 + b^2}{\sqrt{1 + a^2 + b^2}} \right),$$

and $$\left(-\frac{a}{\sqrt{1 + a^2 + b^2}},\ -\frac{b}{\sqrt{1 + a^2 + b^2}},\ -\frac{a^2 + b^2}{\sqrt{1 + a^2 + b^2}} \right).$$

The plane of the ellipse makes the angle $\cot^{-1} \sqrt{a^2 + b^2}$ with the axis of the cylinder and passes through the origin.

It is incidentally proved here that the length of the curve

$$y = a \cos x + b \sin x, \qquad (-\pi \leqslant x \leqslant \pi)$$

is equal to that of an ellipse whose principal axes are of lengths $2 \sqrt{1 + a^2 + b^2}$ and 2.

The curves $y = \sin x$ and $y = \cos x$ roll into plane ellipses whose axes are $2 \sqrt{2}$ and 2, and eccentricity $1/\sqrt{2}$. The planes of these ellipses make angles of 45° with the axis of the cylinder.

Govt. College, Hoshiarpur. HANSRAJ GUPTA.

A proof of a theorem of Wolstenholme.

THEOREM. *If p is a prime > 3, then*

$$\sum_{m=1}^{p-1} \frac{1}{m} \equiv 0 \pmod{p^2} \qquad \ldots \quad (1)$$

1. LEMMA.* *If $1 \leqslant n < p - 1$, then*

$$S_n = 1^n + 2^n + \ldots + (p-1)^n \equiv 0 \pmod{p}.$$

We have $\quad (x+1)^n - x^n = {}^nC_1 x^{n-1} + \ldots + {}^nC_{n-1} x + 1$

whence

$$p^n - 1^n = \sum^{p-1} \{ (x+1)^n - x^n \} = {}^nC_1\, S_{n-1} + \ldots + {}^nC_{n-1} S_1 + (p-1)$$

so that $\quad {}^nC_1\, S_{n-1} + \ldots + {}^nC_{n-1} S_1 \equiv 0 \pmod{p} \qquad \ldots \quad (2)$

The required result follows by mathematical induction from (2) since $S_1 \equiv 0 \pmod{p}$ and ${}^nC_r \not\equiv 0 \pmod{p}$ for $n < p$ $(1 \leqslant r \leqslant n-1)$.

2. Now (1) is equivalent to

$$\sum_{m < \frac{1}{2}p} \left(\frac{1}{m} + \frac{1}{p-m} \right) \equiv 0 \pmod{p^2}$$

or to

$$\sum_{m < \frac{1}{2}p} \frac{1}{m(p-m)} \equiv 0 \pmod{p}$$

or to

$$\sum_{m < \frac{1}{2}p} \frac{1}{m^2} \equiv 0 \pmod{p} \qquad \ldots \quad (3)$$

since

$$\frac{1}{m^2} + \frac{1}{m(p-m)} \equiv 0 \pmod{p}.$$

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* The lemma states a well-known result. The proof given is due to Poinsot, See Dickson : *History of the Theory of Numbers*, Vol. I,

Also by Fermat's Theorem, $m^{p-1} \equiv 1 \pmod p$ for $1 \leqslant m < p$. Hence (3) is equivalent to

$$\sum_{m<\frac{1}{2}p} \frac{m^{p-1}}{m^2} = \sum_{m<\frac{1}{2}p} m^{p-3} \equiv 0 \pmod p \quad \ldots \quad (4)$$

Now since $p-3$ is even, $m^{p-3} \equiv (p-m)^{p-3} \pmod p$, and hence (4) is equivalent to

$$\sum_{m<p} m^{p-3} \equiv 0 \pmod p \quad \ldots \quad (5)$$

By our lemma (5) is true if $p > 3$. Hence (1) is true.

S. CHOWLA and A. SREERAMA SASTRI.

On sums of fifth powers.*

An example of a non-trivial solution in integers of

$$y^4 = x_1^4 + x_2^4 + x_3^4 + x_4^4$$

is due to Norrie (1911).

I find that the sum of five positive fifth powers can be a fifth power. My example is

$$107^5 = 100^5 + 80^5 + 57^5 + 43^5 + 7^5.$$

I also find that the equation

$$x_1^5 + x_2^5 = y_1^5 + y_2^5 + y_3^5 + y_4^5$$

has infinitely many non-trivial solutions in positive integers.

S. SASTRY.

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* A detailed paper on this topic is being prepared for communication to the *London Mathematical Society*.

ANNOUNCEMENTS AND NEWS.

Readers are invited to contribute to the value of this section by sending suitable news items of interest.

The following gentlemen have been elected members of the Society :—

V. G. Gole, Esq., LL.B., Professor, Holkar College, Indore.

Lalchand Gupta, Esq., B.S., M.A., Teacher, Government College, Ludhiana.

M. K. Harihara Iyer, Esq., B.A. (Hons.), Lecturer, St. Joseph's College, Bangalore.

P. Narayanachar, Esq., B.A. (Hons.), Teacher, Municipal High School, Proddatur.

D. Sankaranarayanan, Esq , B.A., L.T., Asst. Meteorological Officer, Poona.

P. C Shah, Esq., B A., Teacher, Proprietory High School, Ahmedabad.

We have great pleasure in including in this issue a portrait (1910) of Prof. G. A. Miller of the University of Illinois, an Honorary Member of the Society for several years. The student world, in particular, owes a deep debt of gratitude to him for a very valuable *Historical Introduction to Mathematical Literature* and a helpful treatise on *Group Theory*. Prof. Miller's critical notes on the History of Mathematics have been a welcome feature of the *Journal*.

Members of the Society, and subscribers to the STUDENT who have not paid their dues for the year are requested to remit their subscriptions to the Treasurer, S. R. Ranganatham, Esq., Librarian, Madras University Library, Senate House, Madras.

The Jubilee Memorial volume of the Society is under preparation and is expected to appear early in 1934. It contains a report of the conference proceedings and full texts of the papers communicated. The price for the volume is Rs. 10. It will be issued free to all members of the Society.

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13. The Theory of Groups is, so to say, entire mathematics divested of its matter and reduced to pure form.

H. POINCARE.

14. The critical study of the history of mathematics is an important element in the preparation of the mathematics teacher, both because many of the statements in our histories are not true and also because a careful study of evidences which seem to be discordant is a useful training for all.

G. A. MILLER.

Owing to the prevalence of plague in Poona, the question of holding the next session of the Science Congress in Bombay is being considered.

The Fourth International Congress of Applied Mechanics will be held at Cambridge, England, on July 3—9, 1934. Promises of manuscripts may be addressed to the Applied Mechanics Division of the American Society of Mechanical Engineers, 29, West 39th Street, New York City.

The elements of Analytical Geometry and the Calculus have been included in the syllabus for the Intermediate Examination of the Osmania University, and the Professors have been requested to write suitable text-books in Urdu.

The Nagpur University has instituted Honours courses in all subjects. The first examination will be held in 1936.

The first examination for the B.Sc. Honours degree in Mathematics under the new regulations of the Annamalai University is to be held in March 1934. The revised courses include a study of transfinite cardinals, elements of group theory, higher plane curves, potential theory, and of vector fields especially in relation to the mathematical theory of electricity and magnetism.

The first of the Intermediate in Arts and the B.Sc. degree examinations of the Annamalai University under the new regulations were held in March 1933. The revised courses for the Intermediate include, in addition, the "analytical geometry of the straight line and the circle" and also the "Elements of the Calculus" (differential and integral); one of the papers for the B.Sc. degree is a 3-hour test in "Statistics."

Among the new "special subjects" in which instruction is now being offered are: "Differential Geometry, including Riemannian Geometry" in the Annamalai University, and "Hydrodynamics" in the Presidency College (Madras University).

According to the revised regulations for the Intermediate in Science examination which are under consideration by the Mysore University, there is to be a third paper in Mechanics and Mensuration for candidates taking mathematics as an optional sebject. There is also to be a paper on "Elementary Mathematics" for those who do not take mathematics as an optional subject. The topics included are: logarithms, quadratic equations, similar figures, mensuration, solution of right-angled triangles in one plane, and statistical graphs including a graphical treatment of measures of deviation and correlation.

Dr. K. S. Krishnan, D.Sc., Secretary of the Indian Association for the Cultivation of Science, has instituted three prizes in the Dacca University to commemorate the services of Sir C. V. Raman, Sir P. C. Ray and S. Ramanujan.

N. G. Shabde, M.Sc., of the City College, Nagpur, has been awarded a scholarship for prosecuting research at Edinburgh with a view to qualifying himself for the D.Sc. degree.

K. Muniruddin, Esq., a graduate in the Osmania University and a teacher in the high school at Hyderabad (Deccan), has returned from Europe after taking the M.A. degree in Mathematics of the Edinburgh University and the Teacher's Diploma in Dublin.

S. Chandrasekharan, Esq., M.A. Madras Government Scholar in Theoretical Physics, has been elected a Fellow of the Trinity College, Cambridge.

Messrs. K. Subba Rao and K. Nagabhushanam are working in the Andhra University as research students in Mathematics.

The Annamalai University has awarded research studentships in mathematics to Messrs. S. Venkatachari (Statistics) and K. Rangaswamy (Geometry).

The University of Madras has awarded the following research studentships in mathematics: Messrs. V. Ganapati (Analysis), A. Somayajulu (Hindu Astronomy), S. Devanathan (Theory of Numbers), N Sundararama Sastry (Statistics), V. Padmanabhan (Hydrodynamics).

Professor D. Kosambi of the Fergusson College, Poona, has accepted an invitation from the Annamalai University to deliver a course of lectures on "The mathematical analysis of Space" at Annamalainagar in January 1934.

The Manchester University announces that a group of citizens have raised a fund to enable the University to ofter a temporary home to some of the scholars, exiled from Germany. Professors R. Baer of Halle (Mathematics) and R. Peieris of Leipzig (Physics) have been appointed for two years.

According to a report in *Nature*, a fine meteoric shower was observed on the night of Oct. 19th, which caused alarm in many places in Western Europe. The shower is attributed to Giacobini's comet (period $6\frac{3}{4}$ years) discovered in 1900 which is estimated to have passed at a distance of only 1 million miles from the Earth.

We are glad to announce that issues of the *Quarterly Journal of Mathematics* are now being received in exchange for the publications of the Society.

Another exchange which has been arranged is with the monthly Journal, *Sphinx*, published in French and dealing with mathematical recreations. The Editor, M. Kraitchik, has kindly sent all the issues since January 1933.

The Gesellschaft der Wissenschaften zu Gottingen has agreed to an exchange of its *Nachrichten* (Math.—Phys. Klasse) with the publications of the Society.

Edward Mann Langley, Founder of *the Mathematical Gazette* and Secretary of the Mathematical Association, 1885—1893, died in June, 1933.

Prof. Lichtenstein of Leipzig died in August, 1933.

Dr. Paul Painleve, Member of the Academie des Sciences, and for sometime Professor at the Sarbonne and later Minister of public instruction, Minister of War and twice premier of France, died in October, 1933 at the age of 70.

Question 1575.

(B. RAMAMURTI) :—ABC, A′B′C′ are two cubicovariant triads on a twisted cubic. H_1, H_2 are the Hessian points. The tangent at H_1 meets the osculating plane at H_2 in P and the tangent at H_2 meets the osculating plane at H_1 in Q_1. Show that the planes ABC, A′B′C′, and the osculating planes at H_1 and H_2 form a harmonic axial pencil. Show also that the quadrics through ABC, A′B′C′ and P, Q touch at P and Q, the tangent planes being PH_1H_2 and QHH_2.

Solution by the Proposer.

Choose H_1PQH_2 $(x_0x_1x_2x_3)$ as the tetrahedron of reference. The equation of the cubic may be written*

$$x_0 : x_1 : x_2 : x_3 = t^3 : t^2 : t : 1$$

the vertices H_1, H_2 corresponding to $t = 0$ and $t = \infty$. Hence the two cubicovariant tetrads may be taken to be given by

$t^3 - 1 = 0$ $(t = 1, \omega, \omega^2)$ and $t_3 + 1 = 0$ $(t = -1, -\omega, -\omega^2)$

$$\left.\begin{array}{lllll} A: & 1, & 1, & 1, & 1 \\ B: & 1, & \omega^2, & \omega, & 1 \\ C: & 1, & \omega, & \omega^2, & 1 \end{array}\right\} \qquad \left.\begin{array}{lllll} A': & -1, & 1, & -1, & 1 \\ B': & -1, & \omega^2, & -\omega, & 1 \\ C': & -1, & \omega, & -\omega^2, & 1 \end{array}\right\}$$

The equations to the planes ABC, A′B′C′ are $x_0 - x_3 = 0$, and $x_0 + x_3 = 0$, which obviously form a harmonic axial pencil with $x_0 = 0$, $x_3 = 0$.

Again the quadrics through ABC, A′B′C′, PQ may be written †

$$(x_0^2 - x_3^2) + \lambda\,(x_0x_3 - x_1x_2) = 0$$

which can be shown to touch at the intersections of $x_0 = 0$, $x_3 = 0$ with $x_0x_3 - x_1x_2 = 0$ (*i.e.*) at P and Q.

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* *Cambridge Tracts*, No. 14, p. 4.

† Salmon : *Analytical Geometry of three dimensions*, p. 123.

Question 1581.

(B. RAMAMURTI) :—A, B, C, D are four points in plane (not forming an ortho-centric tetrad) conormal for some conic passing through them. O is the centre of the rectangular hyperbola through them.

Show that

$$\frac{1}{d_{12}^2 + d_{34}^2 - R^2} + \frac{1}{d_{13}^2 + d_{24}^2 - R^2} + \frac{1}{d_{14}^2 + d_{23}^2 - R^2} = 0$$

where $d_{12} = AB$ and so on, and

$$R^2 = OA^2 + OB^2 + OC^2 + OD^2.$$

Solution by V. A. Mahalingam, S. Narayanan, S. Srinivasan and the Proposer.

Let the equation of the rectangular hyperbola referred to its asymptotes be $xy = k^2$. The points A, B, C, D are then represented by kt_r, k/t_r $(r = 1, 2, 3, 4)$. On calculating d_{12}^2, d_{34}^2, etc., and substituting, it is found that

$$\frac{1}{d_{12}^2 + d_{34}^2 - R^2} = -\frac{1}{2k^2 (t_1 t_2 + t_3 t_4)(1 + 1 / t_1 t_2 t_3 t_4)}.$$

Since A, B, C, D do not form an ortho-centre tetrad, $t_1 t_2 t_3 t_4 \neq -1$. Hence it has to be shown that

$$\sum \frac{1}{t_1 t_2 + t_3 t_4} = 0, \text{ i.e., that } \Sigma t_1 \ \Sigma 1/t_1 = 4 \qquad \ldots \quad (A)$$

Let $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ be a conic through A, B, C, D the normals at which are concurrent. We know that the asymptotes of the rectangular hyperbola, that is, the co-ordinate axes, are parallel to the axes of the conic, and hence separate harmonially the asymptotes of the conic. So the asymptotes of the conic are parallel to $(y + mx)(y - mx) = 0$. Hence $h = 0$.

If the conic passes through kt, k/t, we have

$$ak^2t^4 + + 2gkt^3 + ct^2 + 2fkt + bk^2 = 0,$$

the roots of which are t_1, t_2, t_3, t_4. Hence

$$\sum t_1 = -\frac{2g}{ak} \text{ and } \sum \frac{1}{t_1} = -\frac{2f}{bk} \qquad \ldots \quad (B)$$

Now the centre (x, y) of the conic is given by $ax + g = 0$, $by + f = 0$, and lies on the hyperbola $xy = k^2$. Eliminating x, y, we have

$$gf = abk^2. \qquad \ldots \quad (C)$$

From (B) and (C) it follows that $\Sigma t_1 \Sigma \frac{1}{t_1} = \frac{4gf}{abk^2} = 4$

Question 1589.

(V. RAMASWAMY AIYAR, M A.) :—ABCD is a given parallelogram. A pair of lines drawn through A, which are isogonals in respect to angle A, cut BC in M, M′ and CD in N, N′. Show that the points M, N, M′, N′ are concyclic ; and that as the isogonals vary, the circles MNM′N′ form a co-axal system having A for a limiting point and BD for radical axis.

Solution by A. Ranganatha Rao, K. Rangaswami, M. K. Narasimhamurti, S. Srinivasan, G. L. Pophale, S. Narayanan, T. Manivannan, M. S. Srinivasachari. V. Sundaram and Hansraj Gupta.

Let AMN be in one line. We have

$$\angle NMM' = \angle DAN = \angle M'AB = M'N'C.$$

Hence M, N, M′, N′ are concyclic.

Since, M′AB and AMB are similar triangles

$$\frac{BM}{BA} = \frac{BA}{BM'}, \quad i.e., \quad BM.BM = BA^2.$$

Therefore, the circle MNM′N′ is orthogonal to the circle whose centre is B and whose radius is BA.

Similarly from the similar triangles AND, N′AD, we have $DN.DN' = DA^2$. Hence the circle MNM′N′ is orthogonal to the circle whose centre is D and radius DA.

Thus the circle MNM′N′ belongs to a co-axal system whose radical axis is BD, and one of whose limiting points is A.

Solution by analytical methods by C. Chandrasekhar, and N. K. Narayanamurti.

Question 1599 (Corrected).

(T. R. RAGHAVA SASTRI):—Given a triangle ABC, four conics can be drawn passing through the vertices and having one of their foci at a point P. If e_1, e_2, e_3, e_4 are their eccentricities, prove that

(i) $e_1^2 + e_2^2 + e_3^2 + e_4^2$ will be a minimum if P is the symmedian point of the triangle ABC ;

(ii) if $e_1^2 + e_2^2 + e_3^2 + e_4^2$ is a given constant, the locus of P will be a circle having its centre at the symmedian point.

Solution by S. Narayanan, T. Manivannan, M. S. Srinivasachari and the Proposer.

By reciprocation with respect to a circle with centre P, the conics become circles touching four lines. Hence there are four such conics.

Again, since the conics pass through B and C, if the bisectors of the $\angle$BPC cut BC in points L, L′ and the directrices must pass through L or L′. Taking also the intersections of the bisectors of the angles CPA and APB with CA and AB, we thus obtain six points L, L′, M, M′, N, N′ which lie in threes on four lines. These are the directrices of the four conics. Taking ABC as the reference triangle, the equations of the four directrices may be written

$$\pm px \pm qy \pm rz = 0,$$

where, without loss of generality, we may take p, q, r to be the actual distances of A, B, C from the lines, so that

$$4\triangle^2 = \sum_{p,q,r} a^2 (p - q)(p - r).$$

If p', q', r' are the distances of P from A, B, C, we have

$$p', q', r' = e_1 p, e_1 q, e_1 r,$$

where e_1 is the eccentricity of the conic with directrix $px + qy + rz$, so that

$$4\triangle^2 e_1^2 = \Sigma a^2 (p' - q')(p' - r') \qquad \dots \quad (1)$$

We have also three other relations of type (1) obtained by replacing e_1 by e_2, e_3, e_4 and changing the signs of one the quantities p', q', r'. Adding all these, we get

$$4\triangle^2 (e_1^2 + e_2^2 + e_3^2 + e_4^2) = \Sigma a^2 p'^2 = \overline{PK}^2 \Sigma a^2 + \Sigma a^2 \overline{KA}^2$$

where K is the centroid of masses a^2, b^2, c^2 at A, B, C, that is, the symmedian point of ABC.

Both the given results follow directly from this relation.

Question 1616.

(K. J. SANJANA, M.A.) :—A variable chord of the ellipse

$$E \equiv a^2b^2 - b^2x^2 - a^2y^2 = 0$$

is of given lengh $c < 2b$ and is divided into two parts lc, mc in the constant ratio $l : m$ where $l + m = 1$.

(1) Prove that the locus of the point of division is

$$a^2y\,(lmE - l^2m^2b^4c^2)^{\frac{1}{2}} + b^2x\,(l^2m^2a^2c^2 - lmE)^{\frac{1}{2}} = \tfrac{1}{2}\,(l-m)\,(a^2-b^2)^{\frac{1}{2}}\,E\,;$$

(2) Deduce the equation of the locus of the middle point of the chord ;

(3) Prove further that in both cases the area enclosed by the locus is $\pi\,(ab - lmc^2)$.

Solution by V. A. Mahalingam.

Let the equation to a chord of the ellipse through the point $(x_1 y_1)$ be

$$\frac{x - x_1}{l'} = \frac{y - y_1}{m'} = r \text{ where } = l' \cos\theta \text{ and } m' = \theta,$$

The distances of the points in which the chord meets the ellipse are given by the roots of

$$r^2\,(b^2l'^2 + a^2m'^2) + 2r\,(b^2l'x_1 + a^2m'y_1) - E_1 = 0$$

where $$E_1 = a^2b^2 - b^2x_1^2 - a^2y_1^2.$$

If the segments of the chord measure lc, mc, the algebraic sum of the roots is

$$-\frac{2\,(b^2l'x_1 + a^2m'y_1)}{b^2l'^2 + a^2m'^2} = (l-m)\,c \qquad (1)$$

and the product

$$\frac{-E_1}{b^2\,l'^2 + a^2\,m'^2} = -\,lmc^2. \qquad (2)$$

Hence $$b^2\,l'^2 + a^2\,m'^2\,\frac{E_1}{c^2\,lm}. \qquad (3)$$

Also $$l'^2 + m'^2 = 1. \qquad (4)$$

From (3) and (4)

$$(a^2 - b^2)\, l'^2 = \frac{a^2c^2lm - E_1}{c^2lm}; \quad (a^2 - b^2)\, m'^2 = \frac{E_1 - b^2c^2lm}{c^2lm}.$$

From (1) and (2), we have

$$b^2 l' x_1 + a^2 m' y_1 = - E_1 (l - m) / 2clm.$$

Substituting for l' and m', and dropping the suffixes, we obtain the locus of x, y in the form given in the question.

(2) Let $l = m = \frac{1}{2}$. The equation of the locus of the middle point of the chord reduces to

$$a^4y^2 (4E - b^2c^2) = b^4x^2 (a^2c^2 - 4E).$$

(3) By Holditch's Theorem (*vide* Williamson's *Integral Calculus*, page 207) the area enclosed between the ellipse and the locus (1) is πlmc^2. Hence the area enclosed by the locus (1) is $\pi (ab - lmc^2)$.

Also solved by S. Narayanan and S. Minakshisundaram.

Question 1620.

(K. J. SANJANA, M.A.) :—Find the form of the function $f(x)$ which satisfies the differential equation

$$[f(x)]^2 - 2 \cos A\, f(x) + 1 = a f'(x),$$

where A and a are constants.

Solution by S. Narayanan and T. Manivannan.

Let $y = f(x)$. The equation may then be written

$$\frac{a\,dy}{(y - \cos A)^2 + \sin^2 A} = dx.$$

Integrating, we have

$$\frac{a}{\sin A} \tan^{-1} \frac{y - \cos A}{\sin A} = x + C$$

where C denotes an arbitrary constant.

$$\therefore \quad y = f(x) = \cos A + \sin A \tan (x \sin A/a + c).$$

Question 1622.

(A. RANGANATHA RAO) :—The normals at four points A, B, C, D on a conic S are concurrent at O.

(1) Prove that the circum-centres of ABC, BCD, CDA, DAB lie on a rectangular hyperbola whose asymptotes are the principal axes of S.

(2) If D is a fixed point and O a variable point on the normal at D, the circle ABC generates a co-axial system whose radical axis is a tangent to S.

Solution by S. Narayanan.

If O be (h, k) and if one of the feet of the four normals, say D, be $(x'\ y')$ then A, B, C and $(-x', -y')$ lie on Joachimsthal's circle :

$$x^2 + y^2 + xx' + yy' - u\left(\frac{xx'}{a^2} + \frac{yy'}{b^2} + 1\right) = 0$$

where $u = a^2 + b^2k/y' = b^2 + a^2h/x'$, (*vide* Casey's, *Analytical Geometry*, page 219).

The centre of the circle ABC is

$$(-b^2x'k/2a^2y',\ -a^2y'h/2b^2x').$$

Therefore the centre lies on the rectangular hyperbola $xy = hk/4$ whose asymptotes are the principal axes of S. Hence the result (1).

If $(x'\ y')$ is constant and (h, k) varies, the equation of the circle ABC may be written

$$x^2 + y^2 + yy'\left(1 - \frac{a^2}{b^2}\right) - a^2 - \frac{kb^2}{y'}\left(\frac{xx'}{a^2} + \frac{yy'}{b^2} + 1\right) = 0.$$

As k varies, this generates a co-axal system of circles, the radical axis of the system being $\frac{xx'}{a^2} + \frac{yy'}{b^2} + 1 = 0$, the tangent to S at $(-x', -y')$.

Also solved by V. A. Mahalingam, T. Manivannan. K. Rangaswami and the Proposer.

Remarks by the Proposer.

It can be easily verified that the circles of the co-axal system of (2) pass through D′, the point on the conic diametrically opposite to D, and T′ the foot of the perpendicular from the centre on the tangent at D′. Also it may be noted that in (1) the rectangular hyperpola is the same for all conics co-axial with S, so long as O is fixed.

Remarks by B. Ramamurti.

The line of centres of the co-axial system (2) is the perpendicular bisector of D′T′. It is interesting to note that this line is a normal to the conic Σ obtained from the given conic S by a uniform contraction about the centre in the ratio 2 : 1. Then the totality of Joachimsthal's circles, grouped as above into co-axal systems, correspond to points on the normals to Σ. Since from any point, four normals can be drawn to Σ, it follows that, ***in a concentric system of circles, there are four Joachimsthal's circles of the conic*** S. Further, if, p'_1, p'_2, p'_3, p'_4 be the feet of the normals to Σ, the corresponding points P'_1, P'_2, P'_3, P'_4 are the points of intersection with S of the four circles, other than the co-normal triads of points. Since p'_1, p'_2, p'_3, p'_4 form a conormal tetrad on Σ, so do P'_1, P'_2, P'_3, P'_4 on S. Thus ***the points of intersection with a conic other than the co-normal triads of the four Joachimsthal's circles in a concentric system form a conormal tetrad on the conic.***

Vide also in this connection Jones' *Algebraic Geometry* (1912), page 334.

Question 1613.

(V. M. TELANG):—Let OA be a diameter of a circle, B an extremity of the perpendicular diameter, and BC a chord equal in length to the radius. Join AC and measure AD = AB. Prove that the angle AOD thus constructed is a very close approximation to 40°.

Remarks by S. Narayanan.

The question is fully worked out by the proposer himself on page 234, Part II, of the *Jour. Ind. Math. Soc.* for August 1932.

QUESTIONS FOR SOLUTION.

Proposers of Questions are requested to send their own Solutions along with their Questions.

1654. (S. CHOWLA) :—Given any $a > 0$, is it true that the numbers $\log(1 + p^{-a})$ where p runs through a finite number of primes are linearly independent, *i.e.*, there is no relation of the form

$$\sum_p C_p \log(1 + p^{-a}) = 0$$

where the numbers C_p are integers?

Prove this result in the special case when a is a positive rational number $\leqslant 1$.

1655. (S. CHOWLA) :—Show that every divisor of $z^2 - 1931$ can be expressed in one of the forms $\pm(x^2 - 1931y^2)$ where x, y, z are integers.

1656. (K. S K. IYENGAR) :—If θ is an irrational number, determine the limit as n tends to infinity of $\sum_1^n \frac{|\sin 2\pi n\theta|}{n}$, and show that if p and q are integers, and q tends to infinity, $\sum_1^n \frac{|\sin 2\pi pn/q|}{n}$ tends to the same limit.

1657 (K. J. SANJANA, M. A.) :—Obtain the numerical value of the expression

$$\cos\frac{5\pi}{37} + \cos\frac{13\pi}{37} + \cos\frac{15\pi}{37} + \cos\frac{17\pi}{37} + \cos\frac{19\pi}{37} + \cos\frac{23\pi}{37} + \cos\frac{29\pi}{37} + \cos\frac{31\pi}{37} + \cos\frac{35\pi}{37}.$$

1658. (A. RANGANATHA RAO) :—Construct a quadrilateral three of whose sides are equal, given in magnitude, the remaining side and the angles adjacent to this side.

1659. (T. R. Raghava Sastri):—ABC is a fixed inscribed triangle in a conic S. From any point O on the conic perpendiculars are drawn on the sides to meet the conic again in L, M, N. O′ is the point of concurrency of the perpendiculars drawn from A, B, C on the corresponding sides of triangle LMN.

(1) Show that the locus of O′ is, in general, a conic S′ circumscribing the triangle ABC.

(2) The conics S and S′ are the same when S is a circle or a rectangular hyperbola.

(3) When S is a circle show that the feet of the perpendiculars from O on the sides of the triangle ABC and the feet of the perpendiculars from O′ on the sides of triangle LMN lie on the same straight line.

(4) When S is a rectangular hyperbola, the six feet of the perpendiculars, mentioned above, the three intersections of the corresponding sides of the triangles ABC and LMN, and the three intersections of OL and O′A, OM and O′B, ON and O′C lie on a circle. Show that Question 1433 is equivalent to this result.

(5) When S is neither a circle nor a rectangular hyperbola show that O′ will lie on S when it is the fourth intersection of the circumcircle of ABC with S. Show also that for this position of O′, O will be at the intersections of S with a certain quartic having biflecnodes at A, B, C

1660. (V. Ramaswami Aiyar):—Two plane triangles ABC, A′B′C′ are in perspective. If A′B′C′ be moved rigidly and without rotation so as to be constantly in perspective with ABC,

(1) The locus of the centre of perspective, as in the body of ABC, is a conic X; and, as in the body of A′B′C′, is a conic X′;

(2) The locus of the centre of the (instantaneous) conic of reciprocation F of the two triangles, as in the body ABC, is the conic X; and, as in the body of A′B′C′ is the conic X′:

(3) The conics X, X′ have their points at infinity identical.

List of Journals received in the Library during August and September, 1933.

1 Acta Mathematica, **60**, 1, 2, **61**, 1, 2.

2 American Journal of Mathematics, **55**, 3 (2 copies).

3 Annals of Mathematics, **34**, 3 (2 copies).

4 Astrophysical Journal, **78**, 1, 2.

5 Bulletin of the American Mathematical Society, **39**, 7.

6 Bulletin of the Calcutta Mathematical Society, **24**, 4, **25**, 1.

7 Buletin des Sciences Mathematiques, **51**, June and July, 1993.

8 Current Science, **2**, 2, 3.

9 Gottinger Nachrichten 1933, (Math. Phys. Klasse) Heft 1, 2, 3.

10 Jahresbericht der dentschen mathematiker Vereinigung, **43**, 1 to 4.

11 Mathematische Annalen, **108**, 5.

12 Monthly Notices of the Royal Astronomical Society, **93**, 8.

13 Philosophical Magazine, **16**, 103, 104, 105.

14 Popular Astronomy, **41**, 7 (2 copies).

15 Proceedings of the Edinburgh Mathematical Sociey, **3**, 4 (2 copies).

16 Proceedings of the Physico-Mathematical Society of Japan **15**, 6, 7.

17 Proceedings of the Cambridge Philosophical Society, **29**, 3.

18 Proceedings of the London Mathematical Society, **35**, 5, 7 (2 copies each).

19 Proceedings of the Royal Society of London, **141**, 844, 845.

20 Quarterly Journal of Mathematies, **4**, 14, 15.

21 Rendiconti del Seminario Mathematico, **4**, 1, 2.

22 Sphinx, January to November, 1933.

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All communications intended for publication should be sent to the Editor, A. NARASINGA RAO, Annamalai University, Annamalainagar, South India. Communications relating to the receipt of the Journal, changes of address, and the purchase of current or back numbers of this journal should be sent to the Assistant Secretary, S. MAHADEVA IYER, Assistant Professor of Mathematics, Presidency College, Madras.

Remittances of membership fees and of subscriptions should be sent to the Treasurer, S. R. RANGANATHAN, Librarian, Madras University Library, Senate House, Madras.